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Edge dislocation in a lamellar inhomogeneity with a slipping interface

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The paper develops the solution of the plane elasticity problem in which a source of internal strain is located inside an infinite lamellar inhomogeneity, by assuming that the lamella–matrix interface does not transmit tangential displacements or shear tractions. This condition is very likely to occur in lamellar or layered composite materials at elevated temperatures. It is found that the elastic field may be given in terms of the source bulk field plus one parameter formed from four elastic constants. The solution is used to calculate the image force on an edge dislocation within the lamellar inhomogeneity. It is shown that for some combination of material constants the force differs remarkably from that calculated in the perfectly-adhering interface case.

1. Introduction

In a previous paper [1] the elasticity problem of an edge dislocation in a lamellar inhomogeneity was solved, and the image force on the dislocation calculated and discussed, by assuming a perfect bond between matrix and lamella. This paper investigates the corresponding problem where the lamella–matrix interface, behaving as a viscous fluid layer, can transmit normal displacements and normal tractions, but not shear tractions.

The interaction of a dislocation with a slipping phase boundary was first investigated by Head [2], who proposed to take the slipping interface as a model of a grain boundary at high temperatures. Since then, circular inclusions with slipping interfaces interacting with concentrated forces, and with edge dislocations, have been studied respectively by Dundurs *et al.* [3] and Dundurs and Gangadharan [4]; Ghahremani [5] considered a spherical inclusion with slipping interface loaded in tension, and Tsuchida *et al.* [6] dealt with a slipping elliptic inclusion undergoing a uniform eigenstrain. It has been generally found that the elastic solution is fundamentally different from those obtained by imposing non-slipping boundary conditions. Moreover, some of the above-mentioned studies were used by Srolovitz *et al.* [7–10] in an attempt to explain the creep behaviour of dispersion strengthened alloys at high temperatures. These authors also suggested [8] that at homologous temperatures above approximately 0.6 the assumption of a sliding interface should be more appropriate than that of an adhering one.

The present paper may contribute to the theory of the high temperature mechanical behaviour of materials having a phase in lamellar form, and to that of composite layered materials, whose technological importance is well known [1, 11]. In section 2 the elasticity problem of an internally stressed slipping lamella is stated, and the boundary conditions are written in complex notation. In section 3 the complete solution of the elasticity problem is worked out. In section 4 the image force on an edge dislocation is calculated, plotted, and discussed by comparing it with the force on a dislocation within an adhering lamella. It is in particular deduced that the mechanical behaviour of a lamella embedded in a harder matrix may sensibly change when the temperature is raised to a point where the interface adherence is lost.

2. The elasticity problem

The two-dimensional model which is the object of the present investigation is sketched in Fig. 1. An infinite elastic strip occupies the region $-a \leqslant y \leqslant a$ (region 2) and borders on the elastic half-planes $y \geqslant a$ (region 1) and $y \leqslant -a$ (region 3). The shear modulus and the Poisson's ratio of region 2 (lamella) will be denoted by G_2 and ν_2, respectively, while G and ν denote the shear modulus and the Poisson's ratio of regions 1 and 3 (matrix). The whole material is assumed to be under generalized plane stress or plane strain conditions.

As explained in the introduction, it is assumed that the interfaces $y = a$ and $y = -a$ are not adhering, i.e. they do not transmit tangential tractions or tangential displacements. Thus, the appropriate boundary conditions may be formally written as follows:

$$\left.\begin{aligned} \sigma_{yy}^{(1)} = \sigma_{yy}^{(2)};\quad \sigma_{xy}^{(1)} = \sigma_{xy}^{(2)} = 0;\quad u_y^{(1)} = u_y^{(2)} \\ \text{for } y = a \\ \sigma_{yy}^{(3)} = \sigma_{yy}^{(2)};\quad \sigma_{xy}^{(3)} = \sigma_{xy}^{(2)} = 0;\quad u_y^{(3)} = u_y^{(2)} \\ \text{for } y = -a \end{aligned}\right\} \qquad (1)$$

where $\sigma_{\alpha\beta}$ $(\alpha, \beta = x, y)$ are the in-plane components of the stress tensor, u_x and u_y are the displacement components, and superscripts specify regions. Now we propose to express the conditions (1) in a form suitable for the application of the complex variable method. First of all we remember that stresses and displacements may be expressed through the complex analytic functions $\varphi(z)$, $\psi(z)$ by the following formulas [12]:

$$\left.\begin{aligned} \sigma_{xx} - \mathrm{i}\sigma_{xy} &= \varphi(z) + \overline{\varphi(z)} - \bar{z}\varphi''(z) - \psi'(z) \\ \sigma_{yy} + \mathrm{i}\sigma_{xy} &= \varphi(z) + \overline{\varphi(z)} + \bar{z}\varphi''(z) + \psi'(z) \\ 2G(u_x + \mathrm{i}u_y) &= \kappa\varphi(z) - \overline{z\varphi'(z)} - \overline{\psi(z)} \end{aligned}\right\} \quad (2)$$

where $z = x + \mathrm{i}y$ ($\mathrm{i}^2 = -1$), $\kappa = 3 - 4\nu$ for plane strain, $\kappa = (3 - \nu)/(1 + \nu)$ for plane stress, and rigid body displacements have been omitted in the last equation. Then, we take into account that, apart from an arbitrary additive constant, the resultant force (X, Y) acting on the boundary between two regions may be written as [12]:

$$Y - \mathrm{i}X = -[\varphi(z) + \overline{z\varphi'(z)} + \overline{\psi(z)}] \quad (3)$$

In terms of this force the boundary conditions (1) may be replaced by:

$$\left.\begin{aligned} &Y^{(1)} = Y^{(2)};\quad X^{(1)} = X^{(2)} = 0;\quad u_y^{(1)} = u_y^{(2)} \\ &\qquad\qquad\qquad\qquad \text{for } y = a \\ &Y^{(3)} = Y^{(2)};\quad X^{(3)} = X^{(2)} = 0;\quad u_y^{(3)} = u_y^{(2)} \\ &\qquad\qquad\qquad\qquad \text{for } y = -a \end{aligned}\right\} \quad (1')$$

By introducing the dimensionless coordinates

$$\xi = x/a \qquad \eta = y/a$$

and the function

$$\Phi(\xi, \eta; \lambda) = a^{-1}[\overline{\varphi(z)} + \lambda\bar{z}\varphi'(z) + \lambda\psi(z)] \quad (4)$$

equations (3) and (2) may be written:

$$\left.\begin{aligned} Y - \mathrm{i}X &= -a\overline{\Phi(\xi, \eta; 1)} \\ 2G(u_x + \mathrm{i}u_y) &= a\kappa\overline{\Phi\left(\xi, \eta; -\frac{1}{\kappa}\right)} \end{aligned}\right\} \quad (5)$$

Finally, comparing (5) and (1′), we obtain the conditions at the slipping interface in the following form:

$$\left.\begin{aligned} \mathrm{Re}[\Phi_1(\xi, 1; 1)] &= \mathrm{Re}[\Phi_2(\xi, 1; 1)] \\ \mathrm{Im}[\Phi_1(\xi, 1; 1)] &= \mathrm{Im}[\Phi_2(\xi, 1; 1)] \\ &= 0 \\ \Gamma\kappa\,\mathrm{Im}\left[\Phi_1\left(\xi, 1, -\frac{1}{\kappa}\right)\right] & \\ = \kappa_2\,\mathrm{Im}\left[\Phi_2\left(\xi, 1, -\frac{1}{\kappa_2}\right)\right] & \end{aligned}\right\} \quad (6)$$

$$\left.\begin{aligned} \mathrm{Re}[\Phi_3(\xi, -1; 1)] &= \mathrm{Re}[\Phi_2(\xi, -1; 1)] \\ \mathrm{Im}[\Phi_3(\xi, -1; 1)] &= \mathrm{Im}[\Phi_2(\xi, -1; 1)] \\ &= 0 \\ \Gamma\kappa\,\mathrm{Im}\left[\Phi_3\left(\xi, 1, -\frac{1}{\kappa}\right)\right] & \\ = \kappa_2\,\mathrm{Im}\left[\Phi_2\left(\xi, 1, -\frac{1}{\kappa_2}\right)\right] & \end{aligned}\right\} \quad (7)$$

where subscripts denote regions, Re( ) denotes the real part, Im( ) the imaginary part, and

$$\Gamma = G_2/G$$

3. Elastic fields

Let us assume that an edge dislocation (or, more generally, a line singularity perpendicular to the xy plane) lies at a position $(0, a\eta_0)$, with $-1 < \eta_0 < 1$ (see Fig. 1). The solution of the elasticity problem (i.e. the determination of the elastic field generated by the dislocation in the whole material) will be constructed by the use of the infinite set of dislocation images $(0, a\eta_k)$, where

$$\eta_k = 2k + (-1)^k\eta_0 \quad (k = 0, \pm 1, \pm 2, \ldots)$$

Since both regions 1 and 3 are free of elastic singularities, and the stresses must vanish at infinity, the following double-series expansions of the complex potentials may be considered [1, 11]:

$$\left.\begin{aligned} \varphi_1(z) &= a\sum_{n=0}^{\infty}\left(A_{n,0}\ln\zeta_{-n} + \sum_{m=1}^{\infty}A_{n,m}\zeta_{-n}^{-m}\right) \\ \psi_1(z) &= a\sum_{n=0}^{\infty}\left(B_{n,0}\ln\zeta_{-n} + \sum_{m=1}^{\infty}B_{n,m}\zeta_{-n}^{-m}\right) \end{aligned}\right\} \quad (10)$$

$$\left.\begin{aligned} \varphi_2(z) &= a\sum_{k=-\infty}^{\infty}\left(C_{k,0}\ln\zeta_k + \sum_{m=1}^{\infty}C_{k,m}\zeta_k^{-m}\right) \\ \psi_2(z) &= a\sum_{k=-\infty}^{\infty}\left(D_{k,0}\ln\zeta_k + \sum_{m=1}^{\infty}D_{k,m}\zeta_k^{-m}\right) \end{aligned}\right\} \quad (11)$$

$$\left.\begin{aligned} \varphi_3(z) &= a\sum_{n=0}^{\infty}\left(E_{n,0}\ln\zeta_n + \sum_{m=1}^{\infty}E_{n,m}\zeta_n^{-m}\right) \\ \psi_3(z) &= a\sum_{n=0}^{\infty}\left(F_{n,0}\ln\zeta_n + \sum_{m=1}^{\infty}F_{n,m}\zeta_n^{-m}\right) \end{aligned}\right\} \quad (12)$$

where

$$\zeta = z/a = \xi + \mathrm{i}\eta \qquad \zeta_k = \zeta - \mathrm{i}\eta_k$$

and commas are used to merely separate subscripts. Henceforth, the following ranges of the subscripts k, m and n will be implied:

$$k = 0, \pm 1, \pm 2, \ldots \qquad m, n = 0, 1, 2, 3, \ldots$$

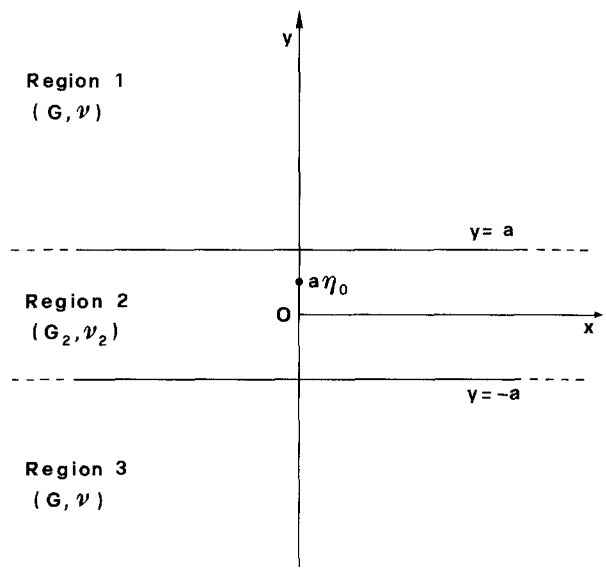

Figure 1 Internally stressed lamellar inhomogeneity.

The complex coefficients $C_{0,m}$ and $D_{0,m}$ yield the elastic field generated by the source in an infinite homogeneous medium of elastic constants G_2 and ν_2 (bulk field). If the bulk field is given, then the problem is reduced to the calculation of the expansion coefficients $A_{n,m}$, $B_{n,m}$, $E_{n,m}$, $F_{n,m}$, $C_{-n-1,m}$, $D_{-n-1,m}$, $C_{n+1,m}$ and $D_{n+1,m}$. This is accomplished by using the boundary conditions (6) and (7).

First we introduce (10)–(12) into (4), obtaining

$$\left.\begin{aligned}
&\Phi_1(\zeta, \eta; \lambda) \\
&= \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} [\bar{A}_{n,m}\bar{\Omega}_{-n,m} + \lambda Q_{n,m}(\eta)\Omega_{-n,m}] \\
&\Phi_2(\zeta, \eta; \lambda) \\
&= \sum_{k=-\infty}^{\infty}\sum_{m=0}^{\infty} [\bar{C}_{k,m}\bar{\Omega}_{k,m} + \lambda M_{k,m}(\eta)\Omega_{k,m}] \\
&\Phi_3(\zeta, \eta, \lambda) \\
&= \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} [\bar{E}_{n,m}\bar{\Omega}_{n,m} + \lambda P_{n,m}(\eta)\Omega_{n,m}]
\end{aligned}\right\} \quad (13)$$

where

$$\left.\begin{aligned}
Q_{n,m}(\eta) &= B_{n,m} - mA_{n,m} \\
&\quad + \mathrm{i}(2\eta - \eta_{-n})\omega_m A_{n,m-1} \\
M_{k,m}(\eta) &= D_{k,m} - mC_{k,m} \\
&\quad + \mathrm{i}(2\eta - \eta_k)\omega_m C_{k,m-1} \\
P_{n,m}(\eta) &= F_{n,m} - mE_{n,m} \\
&\quad + \mathrm{i}(2\eta - \eta_n)\omega_m E_{n,m-1}
\end{aligned}\right\} \quad (14)$$

and

$$\Omega_{k,0} = \ln\zeta_k \qquad \Omega_{k,m} = \zeta_k^{-m} \qquad \text{for } m > 0$$

$$\omega_m = m - 1 - \delta_{1,m} \quad A_{n,-1} = C_{k,-1} = E_{n,-1} = 0$$

$\delta_{1,m}$ being the Kronecker symbol. Substituting (13) into (6) and (7), taking into account that

$$\bar{\Omega}_{k,m} = \Omega_{1-k,m} \qquad \text{for } \eta = 1$$

$$\bar{\Omega}_{k,m} = \Omega_{-1-k,m} \qquad \text{for } \eta = -1$$

and comparing coefficients of the variables $\Omega_{k,m}$ having the same subscripts, we obtain

$$\begin{aligned}
A_{n,m} &= Q_{n,m}(1) \\
&= C_{-n,m} + \bar{M}_{n+1,m}(1) \\
&= \bar{C}_{n+1,m} + M_{-n,m}(1) \\
&= \alpha(C_{-n,m} - \bar{C}_{n+1,m}) \\
E_{n,m} &= P_{n,m}(-1) \\
&= C_{n,m} + \bar{M}_{-1-n,m}(-1) \\
&= \bar{C}_{-1-n,m} + M_{n,m}(-1) \\
&= \alpha(C_{n,m} - \bar{C}_{-1-n,m})
\end{aligned}$$

where

$$\alpha = \frac{\kappa_2 + 1}{\Gamma(\kappa + 1)} \quad (15)$$

Finally, remembering (14), and after some calculation, the following recursion formulas are worked out:

$$\left.\begin{aligned}
&C_{-1,m} = (\alpha + 1)^{-1} \\
&\times [(m + \alpha)\bar{C}_{0,m} - \mathrm{i}(2 + \eta_0)\omega_m\bar{C}_{0,m-1} - \bar{D}_{0,m}] \\
&C_{1,m} = (\alpha + 1)^{-1} \\
&\times [(m + \alpha)\bar{C}_{0,m} + \mathrm{i}(2 - \eta_0)\omega_m\bar{C}_{0,m-1} - \bar{D}_{0,m}] \\
&C_{-n-2,m} = (\alpha + 1)^{-1} \\
&\times [2\alpha\bar{C}_{n+1,m} - 4\mathrm{i}\omega_m\bar{C}_{n+1,m-1} - (\alpha - 1)C_{-n,m}] \\
&C_{n+2,m} = (\alpha + 1)^{-1} \\
&\times [2\alpha\bar{C}_{-1-n,m} + 4\mathrm{i}\omega_m\bar{C}_{-1-n,m-1} - (\alpha - 1)C_{n,m}]
\end{aligned}\right\} \quad (16)$$

$$\left.\begin{aligned}
D_{-1-n,m} &= (m - \alpha)C_{-1-n,m} \\
&\quad + \mathrm{i}(2 + \eta_{-1-n})\omega_m C_{-1-n,m-1} \\
&\quad + (\alpha - 1)\bar{C}_{n,m} \\
D_{n+1,m} &= (m - \alpha)\, C_{n+1,m} \\
&\quad - \mathrm{i}(2 - \eta_{n+1})\omega_m C_{n+1,m-1} \\
&\quad + (\alpha - 1)\bar{C}_{-n,m} \\
A_{n,m} &= \alpha(C_{-n,m} - \bar{C}_{n+1,m}) \\
B_{n,m} &= (m + 1)A_{n,m} - \mathrm{i}(2 - \eta_{-n})\omega_m A_{n,m-1} \\
E_{n,m} &= \alpha(C_{n,m} - \bar{C}_{-1-n,m}) \\
F_{n,m} &= (m + 1)E_{n,m} + \mathrm{i}(2 + \eta_n)\omega_m E_{n,m-1}
\end{aligned}\right\} \quad (17)$$

The set of equations (10)–(12), (16) and (17) solves the problem of determining the elastic field in the whole material when the bulk coefficients $C_{0,m}$ and $D_{0,m}$ are given (convergency is assured by the fact that $|\alpha/(\alpha + 1)| < 1$ for all admissible combinations of material constants.) For an edge dislocation with Burgers vector (b_x, b_y), the solution is completed by the well known relationships:

$$\begin{aligned}
C_{0,0} = \gamma \qquad D_{0,0} = \bar{\gamma} \qquad D_{0,1} = \mathrm{i}\gamma\eta_0 \\
C_{0,m+1} = D_{0,m+2} = 0
\end{aligned} \quad (18)$$

where

$$\gamma = \frac{G_2(b_y - \mathrm{i}b_x)}{a\pi(\kappa_2 + 1)} \quad (19)$$

Equations (16) and (17) show that all the elastic potentials can be expressed in terms of the bulk coefficients plus one parameter, α, formed from four elastic constants. This result is somewhat surprising, because a lamella with an adhering interface [1], as well as finite inhomogeneities with slipping [4] or adhering interfaces, requires two parameters. It is also worth noting that the coefficients given by (16) and (17) do not vanish for $\alpha = 1$. This means that, as expected, an image field generated by a slipping interface exists even when lamella and matrix are made of the same material. Of course, in such conditions no image field is generated by an adhering interface.

4. Force on dislocation

The force (F_x, F_y) acting on the unit length of an edge dislocation located at $z = z_D$, may be expressed through the complex potentials by means of the

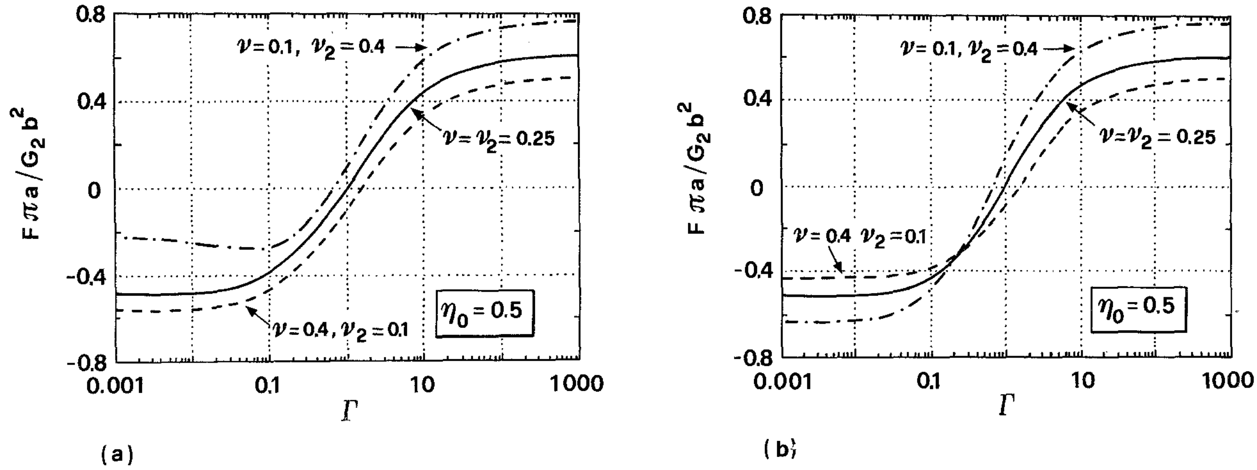

Figure 2 Image force on edge dislocation versus relative hardenss for $b_x = 0$. (a) Adhering interface; (b) slipping interface.

general formula [11]:

$$F_x - \mathrm{i}F_y = b|\gamma| \times \left[\frac{\varphi'_{\mathrm{im}}(z_\mathrm{D}) + \overline{\varphi'_{\mathrm{im}}(z_\mathrm{D})}}{\gamma} + \frac{\bar{z}_\mathrm{D}\,\varphi''_{\mathrm{im}}(z_\mathrm{D}) + \psi'_{\mathrm{im}}(z_\mathrm{D})}{\bar{\gamma}}\right] \tag{20}$$

where the image potentials $\varphi_{\mathrm{im}}(z)$, $\psi_{\mathrm{im}}(z)$ represent the elastic field produced by external loads, including dislocation images. Clearly, in the present case it is

$$z_\mathrm{D} = \mathrm{i}a\eta_0 \tag{21}$$

and the image potentials are obtained by dropping all terms for $k = 0$ in (11):

$$\left.\begin{aligned} \varphi_{\mathrm{im}}(z) &= a\sum_{n=1}^{\infty}\left(C_{-n,0}\ln\zeta_{-n} + \sum_{m=1}^{\infty} C_{-n,m}\zeta_{-n}^{-m}\right) \\ &\quad + a\sum_{n=1}^{\infty}\left(C_{n,0}\ln\zeta_n + \sum_{m=1}^{\infty} C_{n,m}\zeta_n^{-m}\right) \\ \psi_{\mathrm{im}}(z) &= a\sum_{n=1}^{\infty}\left(D_{-n,0}\ln\zeta_{-n} + \sum_{m=1}^{\infty} D_{-n,m}\zeta_{-n}^{-m}\right) \\ &\quad + a\sum_{n=1}^{\infty}\left(D_{n,0}\ln\zeta_n + \sum_{m=1}^{\infty} D_{n,m}\zeta_n^{-m}\right) \end{aligned}\right\} \tag{22}$$

Thus, the image force may be numerically evaluated through (20)–(22), and using (16)–(19) for the calculation of the complex constants $C_{-n,m}$, $D_{-n,m}$, $C_{n,m}$ and $D_{n,m}$.

Since for symmetry reasons F_x vanishes identically, the force is parallel to the dislocation glide direction when $b_x = 0$, or

$$\bar{\gamma} = \gamma$$

In this case Equation (20) reduces to a simpler formula, which may be partially written in explicit form by means of (22), (16) and (17), yielding

$$\begin{aligned} F &= -\gamma b_y \frac{(\alpha-1)}{(\alpha+1)}\left(\frac{1}{1-\eta_0} - \frac{1}{1+\eta_0}\right) \\ &\quad - b_y \operatorname{Im}\sum_{n=2}^{\infty}\left\{\frac{D_{-n,0}}{\mathrm{i}(\eta_0-\eta_{-n})} + \frac{D_{n,0}}{\mathrm{i}(\eta_0-\eta_n)}\right. \\ &\quad \left. - \sum_{m=1}^{n+1}\mathrm{i}^{-m-1}\left[\frac{H_{-n,m}(\eta_0)}{(\eta_0-\eta_{-n})^{m+1}} + \frac{H_{n,m}(\eta_0)}{(\eta_0-\eta_n)^{m+1}}\right]\right\} \end{aligned} \tag{23}$$

where

$$H_{k,m}(\eta_0) = m(D_{k,m} + \mathrm{i}\eta_0\omega_m C_{k,m-1})$$

In other words, the force is the sum of the forces exerted by the slipping interfaces $\eta = 1$ and $\eta = -1$ considered as simply bimetallic (first line of (23)), plus a remainder representing the effect of the higher-order images. Generally such remainder is appreciable, and

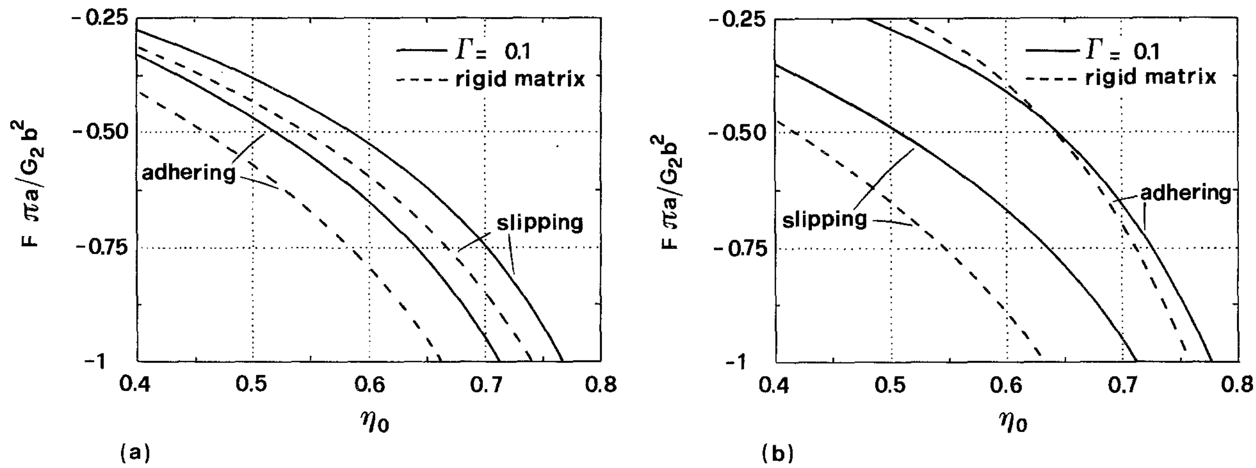

Figure 3 Comparison between adhering and slipping image force on edge dislocation for $b_x = 0$. (a) $\nu = 0.4$, $\nu_2 = 0.1$; (b) $\nu = 0.1$, $\nu_2 = 0.4$.

not less than 20 pairs of images should be included in order to reduce the error below 1%.

Figs 2 and 3 plot the force for plane strain and $b_x = 0$, and have been selected in order to display the differences between the present case and that of a lamella with adhering interface treated in a previous work [1]. Fig. 2a refers to an adhering interface* and shows the force versus the relative hardenss Γ for three Poisson's ratio combinations. Fig. 2b is the analogous plot for a lamella with a slipping interface. No noticeable difference between adhering and slipping interface can be seen for $v = v_2$, whereas a Poisson's ratio mismatch implies important modifications. In Fig. 2a curves corresponding to different mismatches do not intersect. In Fig. 2b the curves intersect for $\Gamma = \Gamma_0$, with $\Gamma_0 \approx 0.2$. This means that (i) for $\Gamma = \Gamma_0$ the force does not substantially depend on the Poisson's ratios; (ii) for $\Gamma > \Gamma_0$ the slipping interface behaves like an adhering one; (ii) for $\Gamma < \Gamma_0$ the trend of the force against the Poisson's ratio mismatch is reversed when an adhering interface becomes slipping. This inversion occurs in whole range of dislocation positions, as confirmed by plots like those shown in Fig. 3.

To sum up, in both cases of adhering and slipping interfaces a dislocation is generally attracted by the nearest interface if the lamella is embedded in a softer matrix ($\Gamma > 1$), whereas it is repelled by a hard matrix ($\Gamma < 1$). Therefore, in the latter case the equilibrium at $\eta_0 = 0$ is stable. If the matrix is hard enough, then a change of the interface from adhering to slipping may sensibly increase (decrease) the restoring force around $\eta_0 = 0$ for $v < V_2 (v > v_2)$, i.e. the lamella may undergo a hardening (softening) process.

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*In Fig. 3 of [1] the labels of the upper and lower curves should be interchanged.